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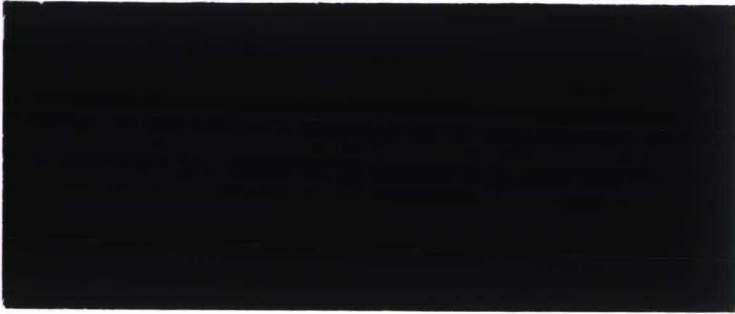
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DEPARTMENT OF ECONOMICS
RESEARCH MEMORANDUM

THE COVARIANCE MATRIX OF ARMA-ERRORS
IN CLOSED FORM

Jan van der Leeuw

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Matrices

Communicated by Dr. H.H. Tigelaar

THE COVARIANCE MATRIX OF ARMA-ERRORS IN CLOSED FORM

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Abstract

Several efficient methods to compute the exact ARMA covariance matrix are known.

However, a general matrix representation in closed form is lacking. This article presents such a closed form. First a matrix equation, containing the covariance matrix, is derived, next it is solved for the MA, AR and ARMA case. The result is quite, and maybe surprisingly, simple.

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1. Introduction

Autoregressive, moving average and mixed processes are widely considered in the statistical literature. Several authors have provided efficient methods of calculating the autocovariance functions (McLeod, (1975), Tunnicliffe Wilson, (1979)). But, although the exact covariance matrix and its inverse of several processes, like AR(1), AR(2), MA(1), ARMA(1,1) are known a general and easy to differentiate form is lacking. Only for the MA(q) case a result, due to Diebold (Diebold, 1986), is known. In this article we present a simple form for the general ARMA(p,q) case, which of course includes the AR(p) and the MA(q) as special cases.

The form of the covariance matrix we present is simple enough to be differentiated, which permits analytical expressions for first and higher order differentials. The results can be used both in time series analysis and in the estimation of the linear regression model with ARMA errors. Furthermore our form gives insight into the way the covariance matrix is composed. As can be expected, the MA covariance matrix is simple when not inverted, the AR part is easy when inverted. The core of the inverted matrix consists of a matrix which rank is equal to the highest number of AR or MA parameters.

2. Matrix form for ARMA parameters

The elements of the ARMA(p,q) error vector ϵ are defined as

$$\epsilon_t = -\sum_{l=1}^p \theta_l \epsilon_{t-l} + v_t + \sum_{l=1}^q \alpha_l v_{t-l} \quad t=1,2,\dots,T \quad (1)$$

where v is a vector of white noise:

$E v_t = 0$, $E v_t^2 = \sigma^2$, $E v_t v_s = 0$ for $t \neq s$. We assume that the ARMA process is stationary over time and that the usual invertibility conditions hold:

$$f(z) = 1 + \theta_1 z + \dots + \theta_p z^p \neq 0 \text{ for } |z| \leq 1 \quad (2)$$

and

$$g(z) = 1 + \alpha_1 z + \dots + \alpha_q z^q \neq 0 \text{ for } |z| < 1 \quad (2')$$

while f and g have no zeros in common.

Following Pagan (1974), we introduce two matrices for both the AR parameters and the MA parameters. These are special types of Toeplitz matrices. First we define a (square) lower band matrix, say P , of dimensions $T \times T$ as follows:

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} = \begin{bmatrix} 1 & & & & & \\ \vartheta_1 & 1 & & & & \\ & \vartheta_1 & \ddots & & & \\ & & \ddots & \ddots & & \\ \vartheta_{p-1} & & & & 1 & \\ \hline \vartheta_p & \vartheta_{p-1} & & \vartheta_1 & 1 & \\ & \ddots & \ddots & \ddots & \ddots & \\ & & \ddots & \vartheta_{p-1} & \vartheta_p & \\ & & & \vartheta_p & \vartheta_{p-1} & \\ & & & & \ddots & \ddots \\ & & & & & 1 \\ & & & & \vartheta_p & \vartheta_{p-1} \\ & & & & & \vartheta_1 & 1 \end{bmatrix}$$

The upper triangular of a lower band matrix consists of zeros and the lower part has off-diagonals with the same elements. As is well-known its inverse can be obtained by a simple algorithm. An other important characteristic of these matrices is the fact that they commute and that their product is a matrix of the same type. It is useful to partition P in P_1 of dimensions $p \times p$, P_2 of dimensions $(T-p) \times p$, P_3 of dimensions $p \times (T-p)$ NE-part (all zero's) and P_4 of dimensions $(T-p) \times (T-p)$.

$$Q = \begin{bmatrix} Q_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \vartheta_p & \vartheta_{p-1} & \ddots & \vartheta_1 \\ 0 & \ddots & \ddots & \\ \vdots & & \ddots & \\ 0 & & & \vartheta_p \\ \hline 0 & & & 0 \\ \vdots & & & \vdots \\ 0 & & & 0 \end{bmatrix}$$

Q has the dimensions $T \times p$ and is partitioned in an upper $p \times p$ part and a lower $(T-p) \times p$ part, which consists of only zeros.

Observe, that P_1 and P_3 have the same structure as P itself. Moreover, P_2 has the same structure as Q , while Q_1 is the transpose of a lower band matrix. In the sequel we will also use the matrices M and N . M and N have the same structure as P and Q with ϑ replaced by α and p replaced by q . Observe that $|P|$ and $|M|$ are equal to 1.

To relate the invertibility condition to these matrices we give the following theorem:

Theorem 1

Let P_1 and Q_1 be defined as above. The invertibility condition is equivalent to the condition that all solutions to $|\lambda P_1 + Q_1| = 0$ satisfy $-1 < \lambda < 1$.

Proof

Observe, that $\lambda P_1 + Q_1$ is a circulant matrix. Its eigenvalues μ_k are (see e.g. Davies, 1979):

$$\mu_k = \lambda + \vartheta_p + \vartheta_{p-1} z_k + \dots + \vartheta_1 z_k^{p-1} \quad (k=1, \dots, p)$$

where $z_k = \lambda^{1/p} e^{2ki\pi/p}$, which implies $|z_k| = |\lambda|^{1/p}$. As $z_k^p = \lambda$, we can also write

$$\begin{aligned} \mu_k &= z_k^p (1 + \vartheta_1 / z_k + \dots + \vartheta_p / z_k^p) \\ &= \lambda f(1/z_k). \end{aligned}$$

f is as defined in (1), where the AR-invertibility condition is stated.

Let λ_0 be a solution to $|\lambda P_1 + Q_1| = 0$ and suppose that the invertibility condition holds. Since λ_0 is never zero we have $\mu_k \neq 0$ for $|1/z_k| \leq 1$ or $|z_k| \geq 1$ or $|\lambda_0| \geq 1$. But $|\lambda_0 P_1 + Q_1|$ can only be zero if at least one of the eigenvalues is zero, which can never be the case for $|\lambda_0| \geq 1$. Therefore we conclude $|\lambda_0| < 1$.

For the second part of the proof, suppose $|\lambda_0 P_1 + Q_1| = 0$ implies $0 < |\lambda_0| < 1$. Then $|\lambda_0 P_1 + Q_1| = 0$ means that at least one of the eigenvalues μ_k is zero or $f(1/z_k) = 0$, while $|\lambda_0| < 1$ means $|1/z_k| > 1$. Hence $f(1/z_k) = 0$ implies $|1/z_k| > 1$, and this is equivalent to $f(1/z_k) \neq 0$ for $|1/z_k| \leq 1$. Q.E.D.

3. Covariance equation

In this section we will derive an equation from which the exact covariance matrix can be solved. First we rewrite the error vector in matrix form. As done by several other authors (de Gooijer, 1978 or Galbraith and Galbraith, 1974) we form an equation for the covariance matrix. But there is one difference as our equation involves only one unknown matrix. The solution to this covariance equation will be given in the next section.

Denoting the covariance matrix by V and using the symbol $-T$ for the inverse of a transposed matrix, we state

Theorem 2

The covariance matrix V corresponding to the ARMA(p,q) error specification is a solution to the equation

$$PVP^T = NN^T + MM^T + [Q \ O]V[Q \ O]^T - [N \ O]M^T P^{-T} [Q \ O]^T - [Q \ O]P^{-1}M[N \ O]^T \quad (3)$$

where P , Q , M and N are defined as above and O is a matrix consisting of zeros.

Proof

First define the auxiliary vectors $\bar{\epsilon}$ and \bar{v} :

$$\bar{\epsilon}^T = (\epsilon_{-p+1}, \epsilon_{-p+2}, \dots, \epsilon_{-1}, \epsilon_0)^T$$

$$\bar{v}^T = (v_{-q+1}, v_{-q+2}, \dots, v_{-1}, v_0)^T$$

Then we can write (1) in matrix form:

$$\begin{bmatrix} Q & P \end{bmatrix} \begin{bmatrix} \bar{\epsilon} \\ \epsilon \end{bmatrix} = \begin{bmatrix} N & M \end{bmatrix} \begin{bmatrix} \bar{v} \\ v \end{bmatrix}$$

or $P\epsilon = N\bar{v} + Mv - Q\bar{\epsilon}$. Post multiplying both sides by its transpose and taking expectations gives

$$PE(\epsilon\epsilon^T)P^T = E(N\bar{v} + Mv - Q\bar{\epsilon})(N\bar{v} + Mv - Q\bar{\epsilon})^T.$$

The right hand side contains the expressions $E\bar{v}\bar{v}^T$, $E\bar{v}v^T$, $E\bar{v}\bar{e}^T$, Evv^T , $Ev\bar{e}^T$ and $E\bar{e}\bar{e}^T$. These can all be expressed in matrix form or are zero. v is an independently distributed variable which implies $Evv^T = \sigma_v^2 I_T$, $E\bar{v}\bar{v}^T = \sigma_v^2 I_p$ and $E\bar{v}v^T = 0$. Because we assume that the ARMA(p,q) process is stationary over time we have the same structure for $E\bar{e}\bar{e}^T$ as for $E\epsilon\epsilon^T$, i.e. V . As the vector \bar{e} depends only on v_0, v_{-1}, \dots (which are by assumption uncorrelated with v_1, v_2, \dots), we conclude $Ev\bar{e} = 0$. The resulting equation can be found in e.g. Galbraith and Galbraith, 1974 or de Gooijer, 1978. But we can go one step further, for the covariances of \bar{e} and \bar{v} have - supposing stationarity - the same structure as the covariances of ϵ and v . This covariance can be derived as follows:

$$\begin{aligned} E(P\bar{e}v^T) &= E(N\bar{v} + Mv - Q\bar{e})v^T \\ &= NE(\bar{v}v^T) + ME(vv^T) - QE(\bar{e}v^T) \\ &= M \end{aligned}$$

which gives $E(\bar{e}v^T) = P^{-1}M$. For $E(\bar{e}\bar{v}^T)$ we get the first p rows and the first q columns of $P^{-1}M$. Using O as the matrix which consists of only zeros gives equation (3).

The problem of finding V is thus reduced to the problem of finding a solution to (3). We will show that this is possible if the invertibility condition holds.

4. Solution of the covariance equation

Theorem 3

The covariance equation (1) has an unique solution if the invertibility condition for the AR-part is fulfilled. The solution is

$$V = [N \ M] [\bar{P}^T \bar{P} - \bar{Q}\bar{Q}^T]^{-1} [N \ M]^T \quad (4)$$

where M and N are as defined in section 2 and \bar{P} and \bar{Q} have the same structure as in section 2, but are of order $(T+p) \times (T+p)$ and $(T+p) \times p$.

Corollary 1

The covariance matrix for the MA(q) model is

$$V=NN^T+MM^T \quad (4a)$$

Corollary 2

The covariance matrix for the AR(p) model is

$$V=[P^T P - Q Q^T]^{-1} \quad (4b)$$

Proof

To prove uniqueness we proceed as follows. Writing (3) in vec-notation and rearranging terms we see that uniqueness is guaranteed if $P \otimes P - [Q \ O] \otimes [Q \ O]$ is not singular. Its determinant, D, is:

$$D = |P \otimes P - [Q \ O] \otimes [Q \ O]|$$

$$= |I - [Q \ O] \otimes [Q \ O] [P \otimes P]^{-1}| |P \otimes P|$$

$$= |I - [Q \ O] P^{-1} \otimes [Q \ O] P^{-1}|.$$

Hence, a sufficient condition for nonsingularity is that all eigenvalues of $[Q \ O] P^{-1}$ are less than one in absolute value. These eigenvalues are zero or equal to those of $Q_1 P_1^{-1}$. As Theorem 1 states that $|\lambda|$ is less than one when the invertibility condition holds, we conclude that D is nonzero, and thus that (3) has an unique solution.

As is proven in the appendix we can write (4) as

$$V = P^{-1} [MM^T + (PN - MQ)(P_1^T P_1 - Q_1 Q_1^T)^{-1} (PN - MQ)^T] P^{-T} \quad (4')$$

That the right hand side of (4') is a solution to (3) is established by direct verification. The essence of the proof is the fact that lower band matrices commute. The proof can be found in the Appendix.

The proof of Corollary 1 is trivial. Substituting $P=I$ and $Q=0$ in (1), we get the MA(q) expression for V.

To prove Corollary 2 substitute $M=I$ and $N=0$, next premultiply both sides of (4') by P and postmultiply by its transpose. The resulting equation is equal to the corresponding covariance equation if V_1 (the NW-part of V) is equal to $(P_1^T P_1 - Q_1 Q_1^T)^{-1}$, which is proven in the appendix. Q.E.D.

It is clear that the second term of (4') within brackets is of order p . Because of the commuting property $PN-MQ$ can be written as $\begin{bmatrix} P_1 N_1 - M_1 Q_1 \\ 0 \end{bmatrix}$, which makes clear that the main part of V consists of $P^{-1}MM^T P^{-T}$, the rest being a correction matrix of which the rank is p . Furthermore (4') is easy to invert: the core of the inverse consists of a $(p \times p)$ matrix, which can be triangulized. Use an expression for the inverse of the sum of two matrices (see *e.g.* Rao, 1973, p. 33), which gives

$$V^{-1} = P^T M^{-T} (I_T - R(R^T R + P_1^T P_1 - Q_1 Q_1^T)^{-1} R^T) M^{-1} P \quad (5)$$

with $R = M^{-1}PN - Q$.

It is not clear whether it is possible to write (5) in a form similar to (4), where the MA part and the AR part are separated.

The determinant of V can be obtained in the following way. Observing that the value of the determinant of $M^{-1}P$ is equal to one we have

$$\begin{aligned} |V| &= |I_T + M^{-1} \begin{bmatrix} P_1 N_1 - M_1 Q_1 \\ 0 \end{bmatrix} (P_1^T P_1 - Q_1 Q_1^T)^{-1} \begin{bmatrix} P_1 N_1 - M_1 Q_1 \\ 0 \end{bmatrix}^T M^{-T}| \\ &= |I_p + (P_1^T P_1 - Q_1 Q_1^T)^{-1} (P_1 N_1 - M_1 Q_1)^T M_1^T M_1 (P_1 N_1 - M_1 Q_1)|. \end{aligned}$$

where M_1 is the $(T \times p)$ matrix, consisting of the first p columns of M^{-1} . The equality is due to the fact, that the second term of the sum in both equations has the same nonzero characteristic roots. The evaluation of the determinant can thus be reduced from a $(T \times T)$ matrix to one of order $(p \times p)$, the highest number of AR or MA parameters.

5. Concluding remarks

In this article we present a compact matrix expression for the covariance matrix of ARMA distributed errors. While the individual elements of the covariance matrix are very complicated, this form is charmingly simple. For the AR case and the MA case the forms are even more simple as can be expected. Expressions for the inverse and the determinant are given.

Furthermore it is shown how the invertibility condition and the positive definiteness of the covariance matrix are interconnected.

Appendix

Because of the structure of the matrices we partition after p rows and columns. We shall use p , the number of AR parameters instead of $\max(p,q)$, because we may suppose p to be equal to q . This gives no loss of generality as it is possible to fill up the shorter vector by zeros. First we will prove the following lemma, next we will show, that (4') is a solution to the covariance equation.

Lemma

$$V=[N \ M][\bar{P}^T \bar{P} - \bar{Q}\bar{Q}^T]^{-1}[N \ M]^T \quad (4)$$

can also be written as

$$V=P^{-1}[MM^T+(PN-MQ)\Delta^{-1}(PN-MN)^T]P^{-T} \quad (4')$$

with $\Delta=P_1^T P_1 - Q_1 Q_1^T$.

Δ is positive definite if the invertibility condition holds.

Proof

First we prove that Δ is positive definite, if the invertibility condition is

fulfilled. Observe, that $\begin{bmatrix} P_1 & 0 \\ Q_1 & P_1 \end{bmatrix}$ and $\begin{bmatrix} Q_1^T & 0 \\ P_1^T & Q_1^T \end{bmatrix}$ are both lower band matrices. As they

commute we have $\Delta=P_1^T P_1 - Q_1 Q_1^T = P_1 P_1^T - Q_1^T Q_1$ or $\Delta=1/2(P_1 P_1^T - Q_1 Q_1^T) + 1/2(P_1^T P_1 - Q_1^T Q_1)$.

Both parts of the right hand side are symmetric, implying that they have real

eigenvalues. Next we show that they are positive. For the first part we have

$P_1 P_1^T - Q_1 Q_1^T = P_1 (I - P_1^{-1} Q_1 Q_1^T P_1^{-T}) P_1^T$. The eigenvalues of the expression between brackets

at the right hand side are equal to one minus the square of the eigenvalues of

$P_1^{-1} Q_1$. But from Theorem 1 we know that $|\lambda P_1 + Q_1| = 0$ implies $|\lambda| < 1$, which means that

an eigenvalue of $P_1 P_1^T - Q_1 Q_1^T$ is equal to $1 - \lambda^2$. In the same way we can prove that the

second part is positive.

To prove that (4) is equivalent to (4') we partition \bar{P} and \bar{Q} as before. For

$\bar{P}^T \bar{P} - \bar{Q} \bar{Q}^T \bar{P}^T \bar{P} - \bar{Q} \bar{Q}^T$ we get $\begin{bmatrix} P_1 P_1^T & Q_1^T P_1 \\ P_1^T Q_1 & P_1^T P_1 \end{bmatrix}$ because $P_1^T P_1 + P_2^T P_2 - Q_1 Q_1^T = P_1 P_1^T$. As is easily

verified, its inverse is $\begin{bmatrix} \Delta^{-1} & -\Delta^{-1} Q_1^T P_1^{-T} \\ -P_1^{-1} Q_1 \Delta^{-1} & P_1^{-1} P_1^{-T} + P_1^{-1} Q_1 \Delta^{-1} Q_1^T P_1^{-T} \end{bmatrix}$. Premultiplying by $[N \ M]$

and postmultiplying by its inverse gives (4'), because P (and thus P^{-1}) and M commute. *Q.E.D.*

To prove **Theorem 3** substitute the right hand side of (4') for V in (3) and partition as before². Observe, that all parts, apart of MM^T , on both sides are zero except the NW part. This means that we have to demonstrate that

$$(P_1 N_1 - M_1 Q_1) \Delta^{-1} (P_1 N_1 - M_1 Q_1)^T = N_1 N_1^T + Q_1 V_1 Q_1^T - N_1 M_1^T P_1^{-T} Q_1^T - Q_1 P_1^{-1} M_1 N_1^T$$

with $V_1 = P_1^{-1} [M_1 M_1^T + (P_1 N_1 - M_1 Q_1) \Delta^{-1} (P_1 N_1 - M_1 Q_1)^T] P_1^{-T}$.

Insert the expression for V_1 , rearrange terms and make use of the commuting property to get $P_1 \Delta^{-1} P_1^T = I + Q_1 \Delta^{-1} Q_1^T$. But this is the NW-part of the covariance-equation in the pure AR-case.

To show this equality, use $\Delta = P_1^T P_1 - Q_1 Q_1^T = P_1 P_1^T - Q_1^T Q_1$ and thus

$$\Delta^{-1} = P_1^{-1} (I - P_1^{-T} Q_1 Q_1^T P_1^{-1})^{-1} P_1^{-T} = Q_1^{-1} (Q_1^{-T} P_1 P_1^T Q_1^{-1} - I)^{-1} Q_1^{-T}.$$

Here we have $P_1^{-T} Q_1 Q_1^T P_1^{-1} = (Q_1^{-T} P_1 P_1^T Q_1^{-1})^{-1}$ and straightforward algebra completes the proof. *Q.E.D.*

² A detailed proof can be obtained from the author upon request.

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